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# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3686

EXPERIMENTAL MEASUREMENTS OF FORCES AND MOMENTS ON A  
TWO-DIMENSIONAL OSCILLATING WING  
AT SUBSONIC SPEEDS

By Sherman A. Clevenson and Edward Widmayer, Jr.

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TWO-DIMENSIONAL OSCILLATING WING  
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## SUMMARY

Experimental results of lifts and moments about the quarter chord of a two-dimensional wing at subsonic Mach numbers are presented. A comparison of the experimental magnitude of the lift vector with the theory as given by Dietze showed good agreement. Comparisons of the moments and the quadrature component of lift with theory indicated that some refinements in the testing technique are necessary for the experimental determination of these quantities in the transonic range.

## INTRODUCTION

An extensive amount of work has been done on the theory of air forces on a harmonically vibrating wing in a two-dimensional compressible medium by Possio, Frazer and Skan, Dietze (references 1 to 4), and others. Flutter of wings in a compressible medium has also been treated by Garrick (reference 5), Frazer and Skan (reference 2), and others in which the aforementioned theoretical air forces are used. Numerous wind-tunnel tests have also been made on the flutter of cantilever wings in the subsonic speed range (reference 6). These tests show reasonable agreement with the indicated theoretical trends. Some work on the experimental determination of the oscillating air forces at high subsonic speeds has been done by Bratt and Chinneck (reference 7), and the oscillating moments on a small airfoil oscillating about the mid-chord position were determined at high subsonic speeds for a range of reduced frequencies of 0.0025 to 0.015. The aerodynamic coefficients have been calculated by Dietze for a reduced-frequency range to  $\omega_r = 0.6$  and for a Mach number range to  $M = 0.7$ .

The present paper deals with some experiments at subsonic speeds in which a two-dimensional wing was oscillated about the quarter-

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<sup>1</sup>Supersedes declassified NACA Research Memorandum L9K28a by S. A. Clevenson and E. Widmayer, Jr., 1950.

chord position in a reduced frequency range from 0.15 to 0.35. Since extension of the theory into the transonic range is extremely difficult and open to considerable doubt because of the occurrence of shocks and mixed flow on the wing, it appears desirable to perform an experiment for determining the oscillating air forces and moments in this region. Although this paper presents few data in the transonic region, it does present an experimental method for obtaining these forces and moments. A comparison is made of the experimental forces and moments with the theory of Dietze. Although comparisons could not be made for all components, it is believed that the presented method can be refined to give sufficient accuracy for the determination of the components of lift and moment of an oscillating airfoil in the transonic range.

#### SYMBOLS

b	semichord
l	representative length (2b)
$\rho$	mass density of gas
v	velocity of gas
Z	unit span of wing
L	oscillating lift ( $L_0 \sin(\omega t + \phi_1)$ )
Q	oscillating moment about quarter chord ( $Q_0 \sin(\omega t + \phi_2)$ )
M	Mach number
t	time
$\alpha$	amplitude of pitching oscillation ( $\alpha_0 \sin \omega t$ )
$\omega_h$	natural circular bending frequency
$\omega_\alpha$	natural circular torsion frequency
$\omega$	circular frequency of oscillation
$\omega_r$	reduced frequency ( $b\omega/v$ )
$\frac{dC_L}{d\alpha}$	slope of lift curve

- $\phi_1$  phase angle between torsional displacement and lift
- $\phi_2$  phase angle between torsional displacement and moment
- $K'_{SD}$  lift coefficient in phase with torsional displacement  

$$\left( \frac{L_o \cos \phi_1}{\pi \rho v^2 b Z \alpha_o} + \frac{\omega_r^2}{2} \right)$$
- $K''_{SD}$  lift coefficient in quadrature to torsional displacement  

$$\left( \frac{L_o \sin \phi_1}{\pi \rho v^2 b Z \alpha_o} \right)$$
- $K'_{DD}$  moment coefficient about quarter chord in phase with torsional displacement  

$$\left( \frac{Q_o \cos \phi_2}{\pi \rho v^2 b^2 Z \alpha_o} + \frac{3}{8} \omega_r^2 \right)$$
- $K''_{DD}$  moment coefficient about quarter chord in quadrature with torsional displacement  

$$\left( \frac{Q_o \sin \phi_2}{\pi \rho v^2 b^2 Z \alpha_o} \right)$$

The subscript  $o$  represents the vector magnitude.

#### APPARATUS

##### Model

The wing has a span of 24 inches and a chord of 8 inches, with an NACA 65-010 airfoil section. It was of fabricated construction, having a steel spar with duralumin skin and was mass balanced about the quarter chord to eliminate any mass-inertia force. This airfoil was supported on bearings at each end and had only freedom to pitch about the quarter-chord point. The wing oscillated about a mean angle of attack of  $0^\circ$  and had a maximum amplitude of  $\pm 2.30^\circ$ . Two variable-frequency, electromagnetic shakers phased to produce pure moment and a torsion rod were attached near one end of the wing (fig. 1). The other end of the torsion rod was fastened to the tunnel wall. Various resonant frequencies could be obtained by the use of torsion rods

having different torsional stiffness. The frequencies of the wing-rod combinations used are as follows:

Wing-rod combination	$\omega_h$ (radians/sec)	$\omega_\alpha$ (radians/sec)
1	528	194
2 (Data not reported)	546	275
3 (Data not reported)	572	361

#### Tunnel

The test section of the Langley  $4\frac{1}{2}$ -foot flutter research tunnel was modified by means of auxiliary walls so that the test section was 24 inches wide. A  $\frac{1}{16}$ -inch gap existed between the model and the tunnel walls. All tests reported were made in Freon-12 in which the speed of sound was about 520 feet per second. The use of this test medium permits the attainment of more than twice the reduced frequency as obtained in air for a given Mach number and torsional frequency. Reynolds numbers for these tests were from  $1 \times 10^6$  to  $5.5 \times 10^6$ .

#### Instrumentation

The instrumentation may be divided into two groups: the first indicating the motion of the wing in the tunnel and its reaction to the flowing gas and the second indicating the supplied power necessary to maintain the oscillation. A 14-channel recording oscillograph was used to record and correlate data from both groups with respect to time.

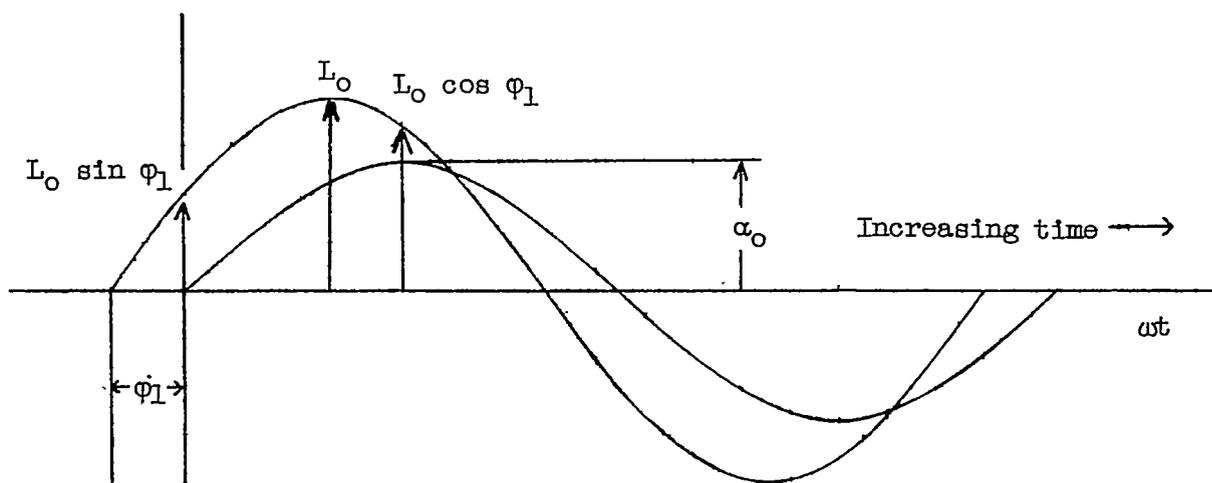
The time history of the pitching motion of the wing was obtained from the signal of an electric strain gage which measured the torsional stress in the torque rod. A light beam, reflected from a mirror fastened to the end of the wing and focused on a translucent scale, also gave an indication of the torsional amplitude of oscillation (fig. 1). Lift was obtained from the signal of electric strain gages mounted on the bearing supports.

The power required to maintain a fixed amplitude of oscillation was measured visually with a wattmeter which was incorporated in the electromagnetic-shaker circuit. At a corresponding time, signals from an ammeter and voltmeter in the same circuit were recorded on the

oscillogram. At the point of wing resonance in pitching, the product of the volts and the amperes was equal to the measured watts.

#### METHOD

The oscillating lifts were measured by strain-sensitive elements on the bearing supports. The incidence of the wing was indicated by the shearing strain in the torsion spring. The phase relation of the lift to the incidence of the wing was obtained from the simultaneous recording of the lift and angular position on an oscillograph. The components of lift and the phase relation to wing incidence are illustrated in the following sketch. The frequency of oscillation is also obtained from the oscillogram.



The aerodynamic damping moment, that is, the moment in phase with the velocity, was obtained by the following procedures. Since a moment does work only with the component that is in phase with the angular velocity, the energy required to sustain an oscillation is a measure of the damping moment and is given by  $\pi Q_0 \alpha_0 \sin \phi_2$ . Therefore the input energy, obtained from the wattmeter and corrected for tare and electric circuit losses, yielded the damping moment. To confirm this value, the aerodynamic damping moment and tare moment were obtained at maximum response ( $\sin \phi_2 = 1$ ) from the ammeter reading and the physical and electric constants of the machine. This aerodynamic moment could then be corrected for the tare damping moment. Two other methods of

measuring damping in common use were considered. If either the logarithmic decrement or the damping from the response curve is used, the aerodynamic damping moment can be obtained. The results of these methods were in agreement.

The aerodynamic moment in phase with the wing position may be obtained experimentally by observing the changes in the natural torsional frequency with Mach number. Such a method was used in reference 7 for an airfoil oscillating about its midchord. Since the model will oscillate in its "damped" natural frequency when the forcing stimulus is removed, the inphase moment may be determined (appendix A). With this knowledge, the shift of the inphase center-of-pressure position from the quarter chord may be determined by dividing by the inphase lift.

#### ACCURACY OF EXPERIMENTAL RESULTS

Before measuring the experimental oscillating lifts, the slope of the experimental static-lift curve of this wing was compared with the lift-curve slope for linear theory as demonstrated in figure 2. The theoretical  $dC_L/d\alpha$  curve is the incompressible slope modified by the

Glauert correction  $\frac{1}{\sqrt{1-M^2}}$ . This corrected lift curve is comparable

to  $K'_{SD}$  for the case of  $\omega_r = 0$  as given by Dietze. Since there was some deviation of the experimental values from the theoretical values for the static case, the values of the experimental data for the oscillating case might also be expected to be less than the values indicated by the linear oscillatory theory.

Quantitative measurements on high-frequency oscillations lead to certain problems not ordinarily encountered in static and low-frequency measurements. Deformation of the model under dynamic loading necessitates the correction of static calibrations. In the case of torsional oscillations the static calibration varied from the dynamic conditions as the result of the difference between the static and dynamic deflection curves. This was corrected by applying a Holzer type of analysis to the dynamic configuration and relating the static strain relative to the midspan angle of incidence to the dynamic strain relative to the midspan angle of incidence. The deformation of the wing obtained from the Holzer analysis was approximately linear, the angle of attack of the side of the wing nearest the shakers being 96.5 percent of the incidence at the opposite end for wing-rod 1. The angle of incidence used in the calculation of the aerodynamic coefficients is that at the midspan.

In the measurement of the oscillatory lifts, the oscillograph records, particularly at the higher Mach numbers, were somewhat

distorted by wind-tunnel vibration and fluctuations in the flow of the air stream. To minimize the error due to this distortion, the oscillograms for both lift and angular position were read by taking an average over five consecutive cycles of an eight-point Fourier analysis. This analysis gave the amplitude of the lift vector and the magnitude of the angle of incidence as well as the phase between the lift and angular deflection. The precision of measurement as determined from repeated analysis from the same record gave lift values within 1 pound, the angle of incidence within  $0.02^\circ$ , and the phase angle within  $\pm 7^\circ$ . Also, it was necessary to correct the phase relationship for any phase difference inherent in the galvanometer string. Unfortunately, the quadrature lift coefficient is very sensitive to variations of phase angle and therefore was subject to error.

The correction due to dynamic magnification on wing-rod 1 was approximately 9 percent of the measured force (see appendix B). No attempt was made to correct the experimental lifts and moments for the aerodynamic effect of this translation, since experiment and analysis showed the maximum displacement at midspan to be in the order of 0.01 inch. This displacement for wing-rod 1 represents less than 1 pound lift (approximately 1 percent total lift) under extreme tunnel conditions of high density and high Mach number.

There was a possibility of the damping moment being somewhat in error due to the variations of the tare damping. Since the magnitude of the tare was from 25 to 60 percent of the measured value, small variations in the tare led to large variations in the aerodynamic-moment coefficients.

A small random error was introduced by the necessity of making interpolations and extrapolations of the theory in order to compare the theory with the experimental results. The interpolation was made by cross-plotting the coefficients against Mach number for a given value of reduced frequency. The extrapolations were made by extending the curves of coefficients against reduced frequency given by Dietze and then cross-plotting the coefficients against Mach number at a given reduced frequency. This procedure tends to admit errors to the theoretical values.

#### IMPROVEMENT OF TECHNIQUE

For the future determination of oscillating air forces, there are various improvements to be effected. For more accurate results, dynamic calibrations for each specific test condition should be made. The difficulty in reading the oscillograms may be ameliorated by increasing the time scale or paper speed of the oscillograms. When

phase angles are being measured, it has been found that using the same type of signal in similarly damped galvanometer elements will reduce any phase difference due to the galvanometers. It is also important to check that the response curves of these elements are "flat" for the frequency range being investigated.

The bending deflection of the wing-rod combination resulted in considerable dynamic magnification of the bearing reactions (see appendix B). The size of the correction is a function of the ratio of the forcing frequency to the natural bending frequency. To keep this magnification as low as possible, wings having very high bending frequencies compared to the forcing frequencies should be used. The tare damping in the system should be a minimum, the evaluation being made at all test amplitudes.

#### DISCUSSION AND RESULTS

The theoretical coefficients of lifts and moments were calculated by using the theory of Dietze (references 3 and 4). However, since the theoretical curves were determined to  $M = 0.7$  only, all reference to theory above that Mach number refers to an extrapolation of these curves. For comparison of the lift coefficients it is well to note that the inphase and quadrature components are functions of both the magnitude and the phase of the lift vector. Under the test conditions it was impossible to treat these factors as independent variables. From figure 3, it may be seen that a comparison can be made between the vector magnitudes of the experimental and theoretical lift coefficients. The difference between the experimental and theoretical values is shown by the short vector marked "difference." It is shown that the experimental  $K'_{SD}$  is insensitive to variations from the theoretical phase angle  $\phi_1$ . Conversely, the  $K''_{SD}$  is highly sensitive to such variations. Since the magnitude of the lift but not the phase angle could be determined accurately, the experimental data which would be in closest agreement with theory would be magnitude of the total-lift vector.

To show the range of values and agreement, the total theoretical lift has been plotted against experimental oscillating lift for wing-rod 1 (fig. 4). The ratio of experimental total lift to theoretical lift was plotted against Mach number (fig. 5). It is clearly shown that the theoretical values are greater than the experimental values. It may be noted that the order of deviation from the theoretical values is of the same magnitude as the deviation of the static-lift data shown in figure 2. In the same graph is plotted  $\omega_r$  showing the range of reduced frequencies in this set of tests. These and other data

pertaining to wing-rod 1, that is, Mach number, coefficients, and so forth, are also shown in table I.

Since the theory assuming an infinite wing in an incompressible flow with some form of compressibility correction is in common usage, it is of interest to compare the experimental data with this theory. The ratio of experimental oscillating lift to this theoretical lift plotted against Mach number is given in figure 6.

A comparison of the experimental phase angle with theory given in figure 7 shows that the trends indicated by theory are followed. When it is remembered that  $\sin \phi_1$  is used in determining  $K''_{SD}$  (see fig. 3), it is only to be expected that the sign and order of magnitude of  $K''_{SD}$  should deviate from the theory. Experimentally determined  $K'_{SD}$  being insensitive to variations in the phase angle permits comparison with theory (fig. 8). It may be noted that the agreement is similar to that obtained for the magnitude of the lift vector.

The opportunity was taken in this series of tests to examine the effects of small amplitudes of oscillation on the force coefficients. It has been suggested that for very small amplitudes some nonlinear effects associated with boundary layer may become significant. For the experiments herein reported no such effect was determined. The effect of various amplitudes of pitching oscillations is shown in figure 9. It can be seen that the ratio of coefficients for the four Mach numbers shown is relatively constant and independent of amplitude, indicating that the amplitude of oscillation had little effect on the lift coefficient.

In view of the large corrections in the lift, angle of incidence, and phase angle necessary in reducing the data for wing-rod 3, it was deemed advisable to omit the presentation of data and comparisons with theory. However, all trends were similar to those of wing-rod 1. Experiments were also conducted on wing-rod 2 which had a natural frequency of 45 cycles per second (midway between wing-rods 1 and 3). The data obtained for this combination (not presented) are subject to question. It was found that the measured value of damping in pitch decreased with Mach number, approaching zero at  $M = 0.7$ . The ratio of the experimental to the theoretical oscillation lift coefficients also approached a minimum at  $M = 0.7$ . The data obtained for wing-rods 1 and 3 imply that this effect may be restricted to the particular test conditions for wing-rod 2. Unfortunately, time was not available to study this problem further. It might be noted that the phenomenon exhibits certain characteristics of a self-excited vibration. The reason for the behavior is unexplained and it is hoped that future studies may answer the problem.

As previously mentioned, the experimental damping-moment coefficient in quadrature with the torsional displacement was determined. A comparison of these experimental results with theory plotted against Mach number (fig. 10) shows the experimental values to be consistently low. This result has also been noted by Bratt and Chinneck (reference 7) for subsonic speeds. Some of the variation in the experimental coefficients may be attributed to fluctuations of the tare damping.

The inphase moment coefficients  $K'_{DD}$  were determined as shown in appendix A. Since, in the present case, the airfoil was oscillated about its quarter chord, the inphase moment was small. Thus, the change in natural torsional frequency may have been within the accuracy of the experiment. Therefore, a comparison for  $K'_{DD}$  of experiment with theory shows little agreement (table I). However, for wing-rod 1 which had the weakest spring and consequently the greatest sensitivity to aerodynamic moments, at the high Mach numbers (0.7 to 0.8) a negative oscillating moment occurred on the airfoil. This would indicate a rearward shift of the inphase center-of-pressure position of about 3 percent chord. This center-of-pressure shift was predicted theoretically, but to a lesser degree (fig. 11).

#### CONCLUDING REMARKS

It is believed that the method of obtaining oscillating lifts and moments due to pitch as described in this paper can be refined to give sufficient accuracy for the experimental determination of those quantities in the transonic region.

Good agreement of the experimental magnitude of the oscillating lift vector was obtained with the magnitude of the theoretical lift vectors of Dietze. Similar agreement was obtained between the experimental coefficients in phase with the wing displacement and the corresponding theoretical values. In general, both experimental coefficients were lower than the corresponding theoretical values.

The trend with Mach number and reduced frequency indicated by theory for the phase between lift and incidence was obtained experimentally. The sensitivity of the quadrature components of lift to the variation in the experimental phase angle was such that the values obtained were questionable.

The amplitude of torsional oscillation apparently has little effect on the lift coefficients in the range of 0.004 to 0.04 radian.

A comparison of the experimental damping-moment coefficients with the theoretical values showed the experimental values to be consistently low. While quantitative agreement of the inphase moment coefficients with the theoretical values is not obtained, the movement of the inphase center of pressure tends to follow the behavior indicated by theory.

Langley Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Langley Field, Va., November 30, 1949.

## APPENDIX A

DETERMINATION OF THE INPHASE MOMENT AND  
THE CENTER-OF-PRESSURE SHIFT

The determination of the inphase moment, when the change of natural frequency of oscillation of a wing is known, is as follows:

The definition of the moment is the air spring times the wing angular deflection, namely  $\Delta K_\theta \alpha_0$ . The air spring is determined as follows: The natural frequency is

$$\omega_1 = \sqrt{\frac{K_\theta}{I}}$$

the frequency with air spring force is

$$\omega_2 = \sqrt{\frac{K_\theta + \Delta K_\theta}{I}}$$

thus

$$\left(\frac{\omega_1}{\omega_2}\right)^2 = \frac{K_\theta}{K_\theta + \Delta K_\theta}$$

then

$$\Delta K_\theta = K_\theta \left[ \left(\frac{\omega_2}{\omega_1}\right)^2 - 1 \right] \quad (A1)$$

Now, since the air spring is known, the inphase moment is easily calculated.

From inspection of equation (A1), when  $\Delta K_\theta$  is positive,  $\omega_2 > \omega_1$  and there is a rearward shift of the center of pressure. When  $\Delta K_\theta$  is

negative,  $\omega_1 > \omega_2$  and there is a forward shift of the center of pressure. The position of the center of pressure referred to the quarter chord is determined by the ratio of the inphase moment to the inphase lift force, namely

$$\text{C.P.} = \frac{\Delta K \theta \alpha_0}{2L_0 \cos \phi_1}$$

This gives the center-of-pressure shift from quarter-chord position in percent chord.

For comparison with theory, the center-of-pressure shift was calculated by the theory of Dietze (reference 3) as follows:

$$\begin{aligned} \text{C.P.} &= \frac{Q}{2L_0 \cos \phi_1} \\ &= \frac{\frac{\pi}{2} \frac{\rho V^2}{2} l^2 \Delta Z \alpha_0 \left( \frac{3\omega_r^2}{8} - K'_{DD} \right)}{2\pi \frac{\rho V^2}{2} l \Delta Z \alpha_0 \left( \frac{\omega_r^2}{2} - K'_{SD} \right)} \\ &= \frac{\frac{1}{2} \frac{3}{8} \omega_r^2 - K'_{DD}}{\frac{1}{2} \frac{1}{2} \omega_r^2 - K'_{SD}} \\ &= \frac{K'_{DD} - 0.375 \omega_r^2}{2K'_{SD} - \omega_r^2} \end{aligned}$$

## APPENDIX B

## DYNAMIC MAGNIFICATION OF LIFT FORCE

When an elastic body is subjected to an oscillating force, it is possible to determine the amplitude response of the resulting vibration. In the case of a simply supported undamped beam carrying a uniformly distributed oscillating load (reference 8) it may be shown that

$$y = y_s \sum_{i=1}^{i=\infty} \frac{\sin \frac{i\pi x}{l} \sin \omega t}{i \left[ i^4 - \left( \frac{\omega}{\omega_r} \right)^2 \right]} \quad (B1)$$

where

$$y_s = \frac{4W_0 l^4}{\pi^5 EI}$$

and  $y_s$  is approximately equal to the static midspan deflection under a uniform load  $W_0 l$ . In view of this relation, and recognizing that the deflection curve is a function of the loading, it follows that the reactions at the supports are also dependent upon the deflection curve. This fact may be shown as follows for a simply supported beam having an arbitrary loading:

$$y = \iint \frac{M}{EI} dx dx$$

$$\frac{d^3 y}{dx^3} = \frac{d \left( \frac{M}{EI} \right)}{dx} = \frac{1}{EI} \frac{dM}{dx} + M \frac{d \left( \frac{1}{EI} \right)}{dx}$$

For constant EI

$$\frac{d^3 y}{dx^3} = \frac{1}{EI} \frac{dM}{dx} = \frac{V}{EI}$$

where V, the vertical shear, is given by  $R - \int \lambda(x) dx$  and  $\lambda(x)$  is span loading.

It may be shown that

$$\lim_{x \rightarrow 0} \frac{d^3 y}{dx^3} = \frac{R}{EI} \quad (B2)$$

where R is the reaction at the support. Using the deflection curve from equation (B1), the reactions for a simply supported, undamped beam subjected to a uniformly distributed oscillating load are given by

$$R = -\frac{4}{\pi^2} W_0 l \sum_{i=0}^{i=\infty} \frac{\sin \omega t}{(2i+1)^2 - \left(\frac{1}{2i+1}\right)^2 \left(\frac{\omega}{\omega_r}\right)^2} \quad (B3)$$

Treatment of this nature roughly approximated the physical conditions of the experiment and gives an indication of the order of the correction. For the higher frequency combinations, a closer approach to actual conditions is obtained by considering the wing-rod as a two-span continuous beam built in at the wall. The reactions of this structure were determined by using the method of moment distribution. By assuming a given distributed load the deflection curve was obtained. The inertia effects  $m\omega^2 y$  were then known for that loading. Using the combination of the distributed and inertia loads a new deflection curve was obtained. By use of successive approximations the total loading was obtained. The ratio of the applied (aerodynamic) load to the measured load is then known.

For the low-frequency wing-rod combination, this analysis, the results from equation (B3), and experimentally determined correction factors were in agreement. The variation in the results of these methods for wing-rod 3, having a frequency ratio  $\omega/\omega_r$  of 0.65, was such that only the order of magnitude of the correction factor could be determined.

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TABLE I  
WING-ROD 1

$\rho$ (slugs/cu ft)	$v$ (fps)	M	$\sigma_f$	$\alpha_0$ (radians)	Lift loads angle of incidence (deg)		Lift vector (lb)		$K'_{ED}$		$K''_{ED}$		$K'_{ID}$		$K''_{ID}$		
					Exp.	Theory	Exp.	Theory	Exp.	Theory	Exp.	Theory	Exp.	Theory	Exp.	Theory	
$562 \times 10^{-6}$	339	0.674	0.1996	0.01977	-6.79	-4.841	41.8	45.35	1.566	1.743	-0.1842	-0.146	-----	0.0413	0.2273	0.315	
	339	.662	.1860	.02466	-2.51	-5.147	47.6	57.09	1.449	1.760	-.6260	-.157	0.0084	.0358	.2261	.294	
	339	.662	.1860	.01937	-.96	-5.147	33.9	45.63	1.317	1.760	-.0218	-.157	.0084	.0358	.2429	.294	
	339	.662	.1860	.01166	-4.18	-5.147	22.85	27.36	1.427	1.760	-.1033	-.157	.0084	.0358	.2413	.294	
	315	.624	.2027	.02234	-2.99	-3.032	39.9	45.99	1.476	1.701	-.0760	-.069	-----	.0292	.1954	.296	
	313	.615	.2003	.01928	-.64	-2.799	32.85	38.68	1.448	1.697	-.0159	-.062	.00937	.0273	.2323	.293	
	313	.615	.2003	.00885	6.74	-2.799	15.96	17.75	1.517	1.697	-.1773	-.062	.00937	.0273	.3432	.293	
	3999	.284	.2254	.03012	7.37	.707	41.8	48.47	1.375	16.45	.1653	.02	-.0244	.0207	.1724	.305	
	6030	.282	.2285	.02175	9.05	1.136	33.2	34.33	1.522	1.640	.2035	.032	-.0246	.0206	.2180	.308	
	6030	.282	.2285	.01393	6.10	1.136	21.75	22.62	1.566	1.640	.1575	.032	-.0246	.0206	.2649	.308	
	6512	1.78	.348	.3536	.04098	22.47	16.47	23.0	25.90	1.264	1.467	.4975	.415	-.0843	.0087	.1776	.400
	6498	1.61	.333	.3479	.03317	23.49	15.85	17.65	21.92	1.143	1.470	.4704	.400	-.0819	.0093	.2626	.396
	6498	1.61	.333	.3455	.01289	15.14	19.68	8.34	8.451	1.956	1.474	.3785	.397	-.0819	.0093	.7273	.393
	1456	408	.786	.03019	-17.3	-20.55	26.61	30.19	1.548	1.920	-.5183	-.355	-.0479	.0780	.1325	.344	
	1559	348	.671	.1866	.0323	-4.2	-5.707	20.35	22.20	1.990	1.773	-.1159	-.178	-.064	.0382	.09737	.291
	1668	283	.547	.2284	.03220	7.85	-6.376	12.1	14.66	1.351	1.637	.1829	.035	-.0831	.0200	.06817	.174
	3679	389.1	.770	.1714	.01762	-8.82	-9.720	36.33	38.96	1.765	1.883	-.2709	-.320	.1297	.0698	.2102	.373
	3679	389.1	.770	.1714	.01326	-9.33	-9.720	26.0	29.22	1.692	1.883	-.2721	-.320	.1297	.0698	.2312	.373
	3679	389.1	.770	.1714	.01041	-10.3	-9.720	22.61	23.14	1.630	1.883	-.3508	-.320	.1297	.0698	.2655	.373
	3793	369	.731	.1868	.02059	-4.6	-7.393	38.7	40.01	1.715	1.805	-.4102	-.232	.1609	.0586	.2004	.373
	3793	369	.731	.1868	.01745	-9.12	-7.393	32.0	33.22	1.683	1.805	-.2672	-.232	.1609	.0586	.2234	.373
	3793	369	.731	.1868	.0155	-4.2	-7.393	26.05	28.13	1.635	1.805	-.4098	-.232	.1609	.0586	.2205	.373
	3793	369	.731	.1868	.01047	-7.45	-7.393	17.25	17.16	1.813	1.805	-.2350	-.232	.1609	.0586	.2773	.373
	3793	369	.731	.1868	.0062	-13.9	-7.393	13.4	13.34	1.776	1.805	-.4346	-.232	.1609	.0586	.2350	.373
	3931	343	.6755	.1885	.02321	-4.23	-5.496	41.8	41.80	1.776	1.764	-.1294	-.168	-.03717	.0402	.1896	.310
	3931	343	.6755	.1885	.01849	-1.25	-5.496	35.3	35.72	1.751	1.764	-.0378	-.168	-.03717	.0402	.1883	.310
	3931	343	.6755	.1885	.01745	-.50	-5.496	28.0	30.96	1.606	1.764	-.0138	-.168	-.03717	.0402	.1929	.310
	3931	343	.6755	.1885	.01344	2.3	-5.496	22.1	25.19	1.930	1.764	.0615	-.168	-.03717	.0402	.2126	.310
	3931	343	.6755	.1885	.01169	-.50	-5.496	16.32	18.05	1.520	1.764	-.0136	-.168	-.03717	.0402	.2066	.310
	4439	208.4	.407	.3141	.03918	13.9	11.66	20.65	23.79	1.316	1.908	.3133	.301	-.1205	.0110	.1017	.368
	4439	208.4	.407	.3141	.03223	15.71	11.66	16.45	19.40	1.264	1.908	.3420	.301	-.1205	.0110	.4175	.368
	4439	208.4	.407	.3141	.02571	19.34	11.66	12.97	15.48	1.225	1.908	.437	.301	-.1205	.0110	.1215	.368
	4439	208.4	.407	.3141	.01757	15.28	11.66	9.87	11.26	1.307	1.908	.3438	.301	-.1205	.0110	.1567	.368
	4439	208.4	.407	.3141	.01044	14.81	11.66	6.08	7.038	1.295	1.908	.4943	.301	-.1205	.0110	.2387	.368
	4439	208.4	.407	.3141	.00588	3.27	11.66	3.48	3.541	1.504	1.908	.2014	.301	-.1205	.0110	.7110	.368
	3679	389.1	.77	.1714	.02086	-5.12	-9.720	38.9	46.86	1.575	1.883	-.1403	-.320	.1297	.0698	.1890	.373
	3679	389.1	.77	.1714	.00815	-11.54	-9.720	12.16	13.89	1.639	1.883	-.3320	-.320	.1297	.0698	.2919	.373
	3793	369	.73	.1868	.01283	-8.90	-7.393	19.92	23.96	1.500	1.805	-.2319	-.232	.1609	.0586	.2377	.373
	3931	343	.675	.1885	.00471	-11.12	-5.496	7.48	6.996	1.898	1.764	-.3594	-.168	-.03717	.0402	.3015	.310
	3931	343	.675	.1885	.00117	-2.40	-5.496	10.82	13.40	1.433	1.764	-.0588	-.168	-.03717	.0402	.2031	.310

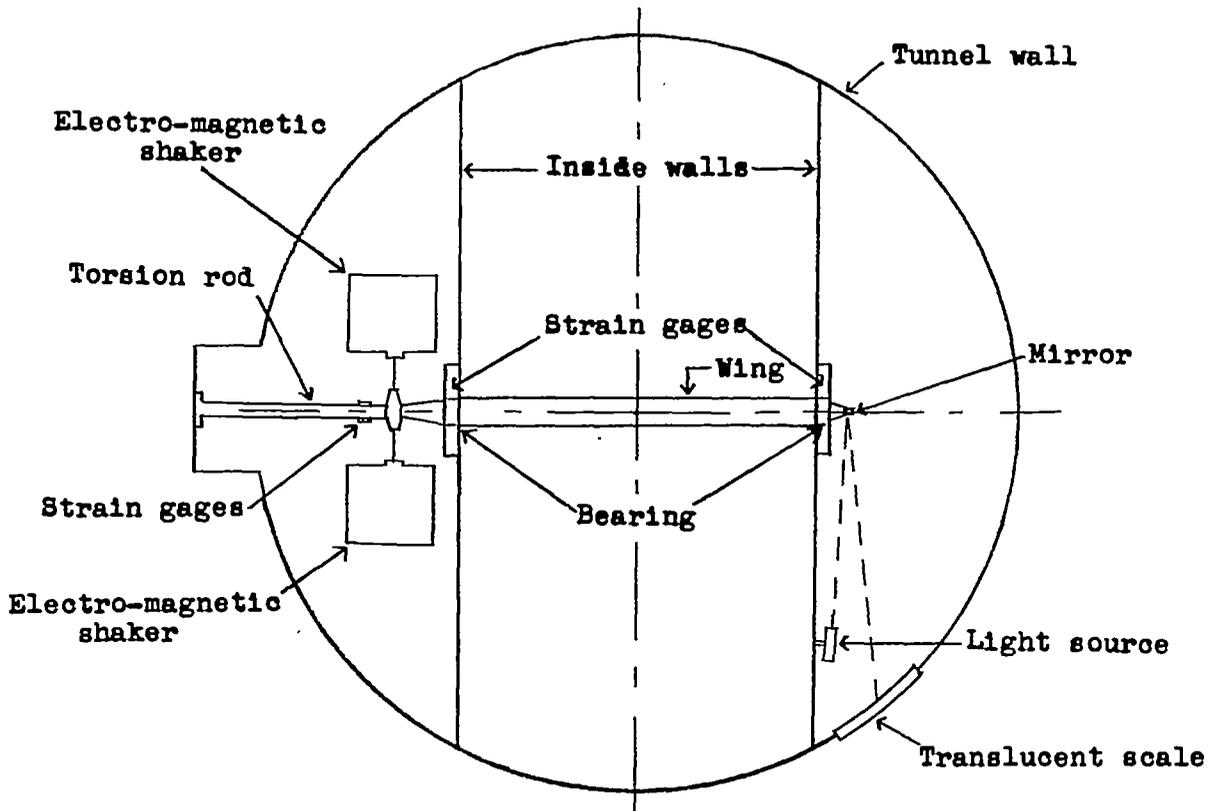


Figure 1.— Diagrammatic view of wing in the Langley  $4\frac{1}{2}$ -foot flutter research tunnel.

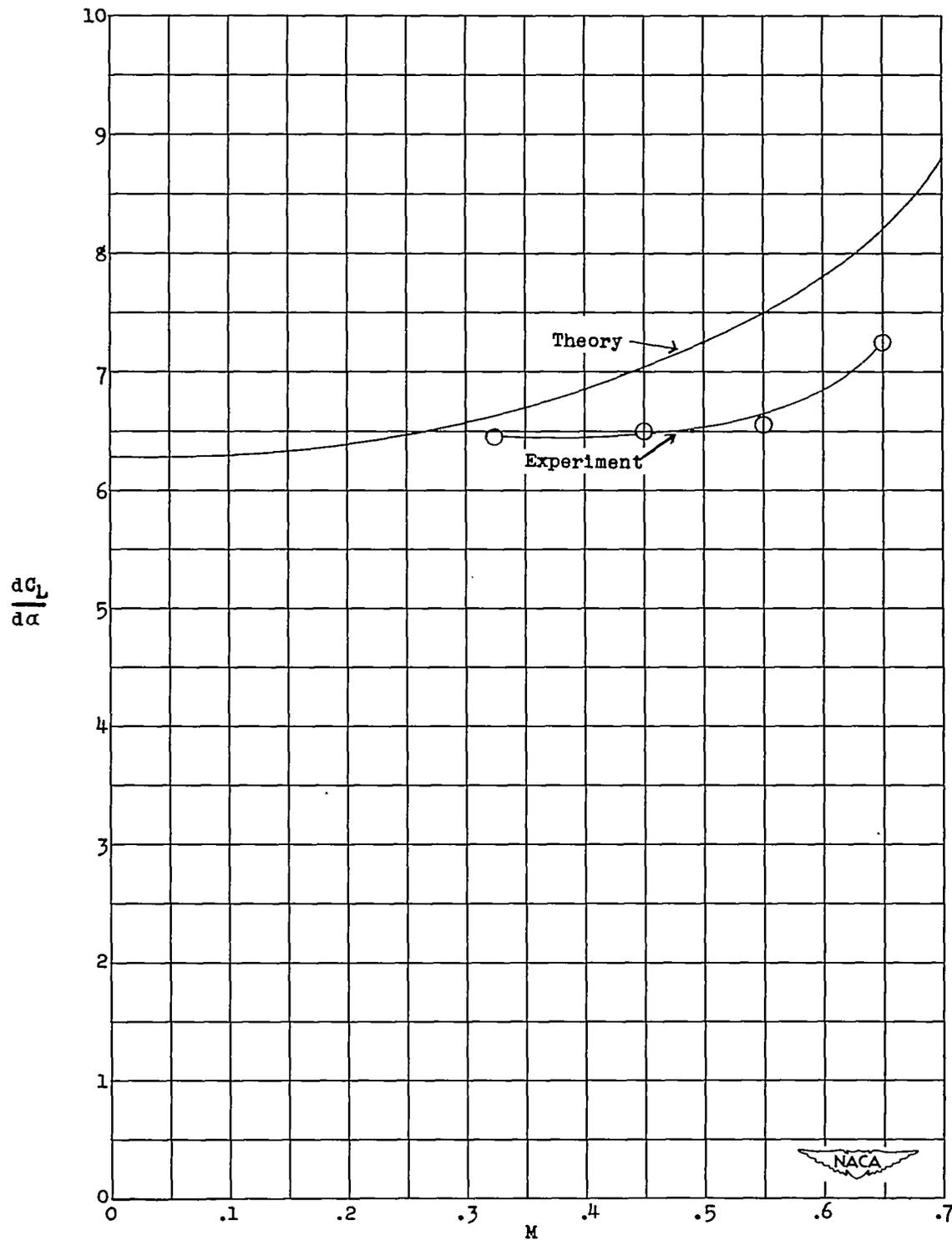


Figure 2.— Slope of the static-lift curve as a function of Mach number for wing-rod 1.

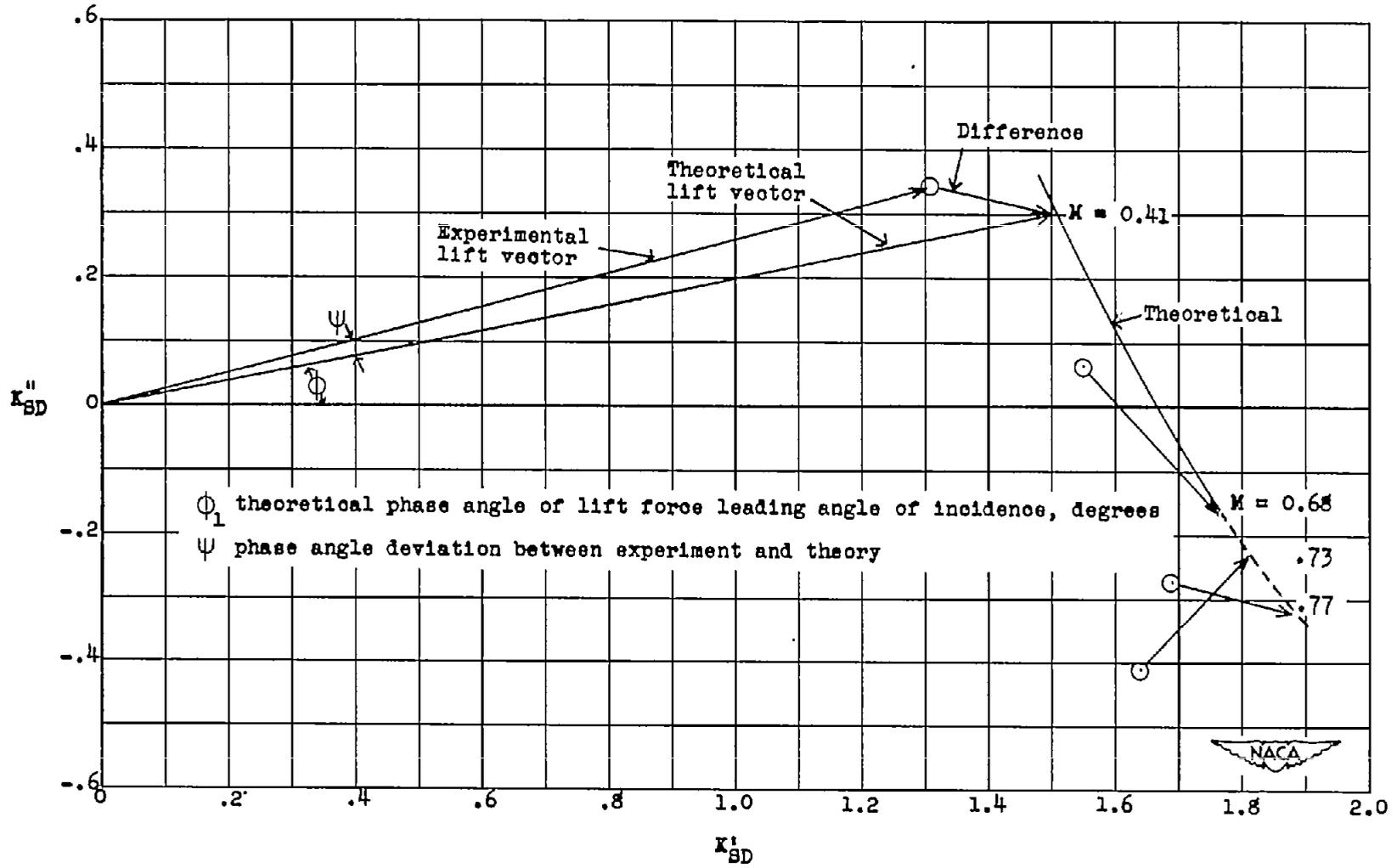


Figure 3.-  $K''_{SD}$  against  $K'_{SD}$  showing comparisons of experimental lift coefficients and phase angle with theory.  $\rho = 0.00393$ ,  $\alpha_0 = 0.013$  for wing-rod 1.

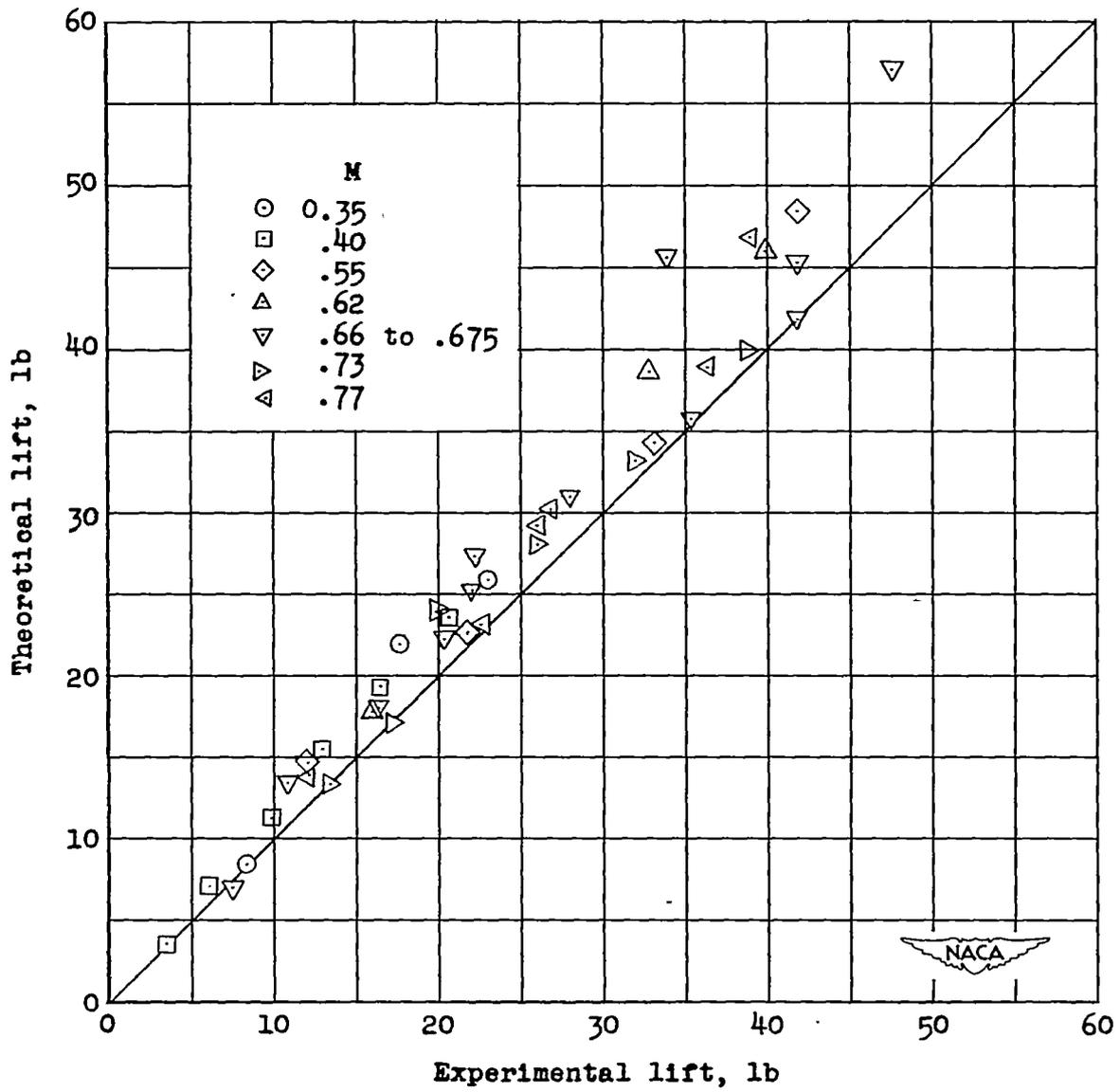


Figure 4.- Theoretical lift against experimental lift for wing-rod 1.



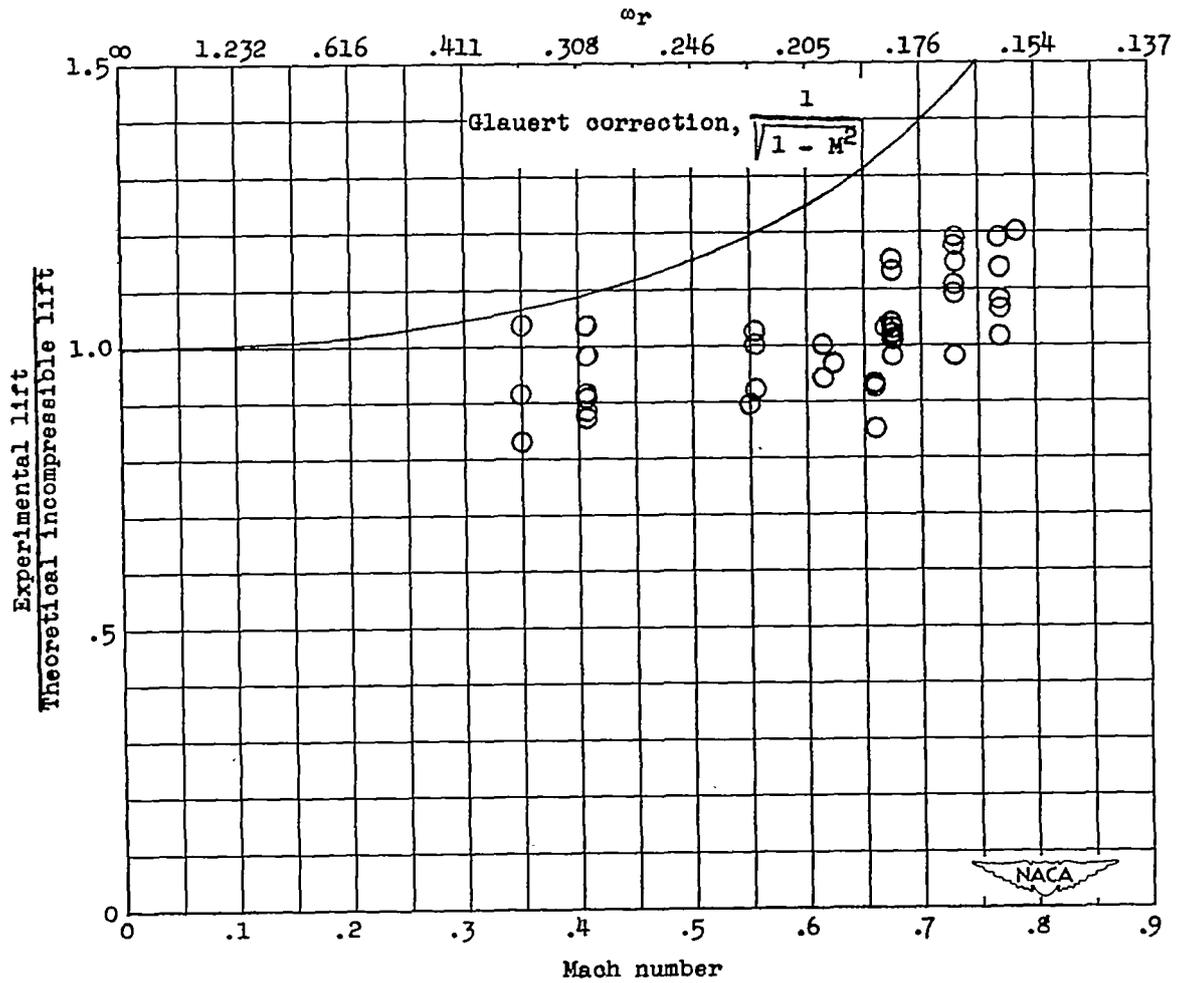


Figure 6.— Ratio of experimental lift to theoretical lift for incompressible flow at various Mach numbers and reduced frequencies for wing-rod 1.

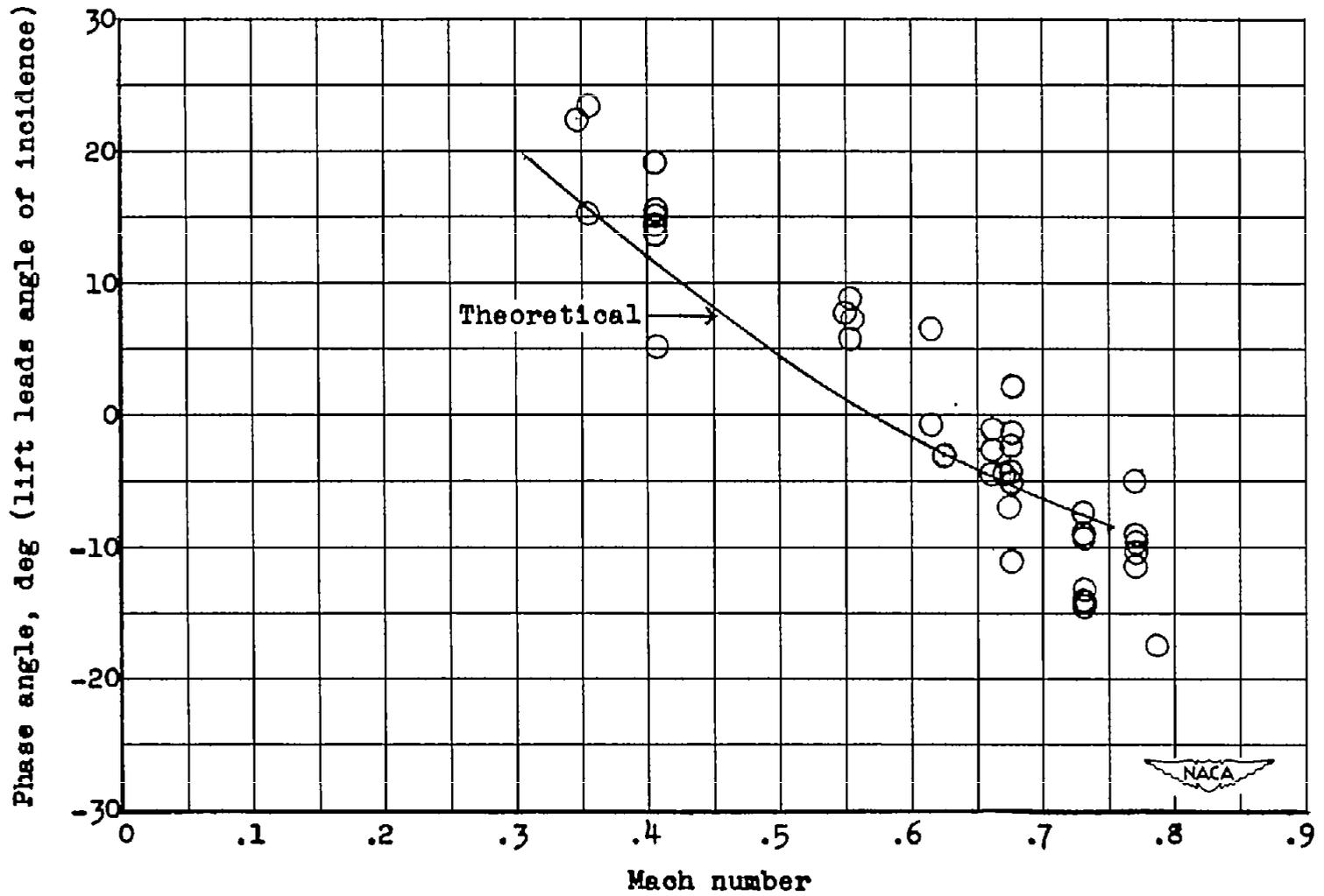


Figure 7.- Theoretical and experimental phase-angle relationship to Mach number for wing-rod 1.  $\omega_{\alpha} = 194$  radians per second.

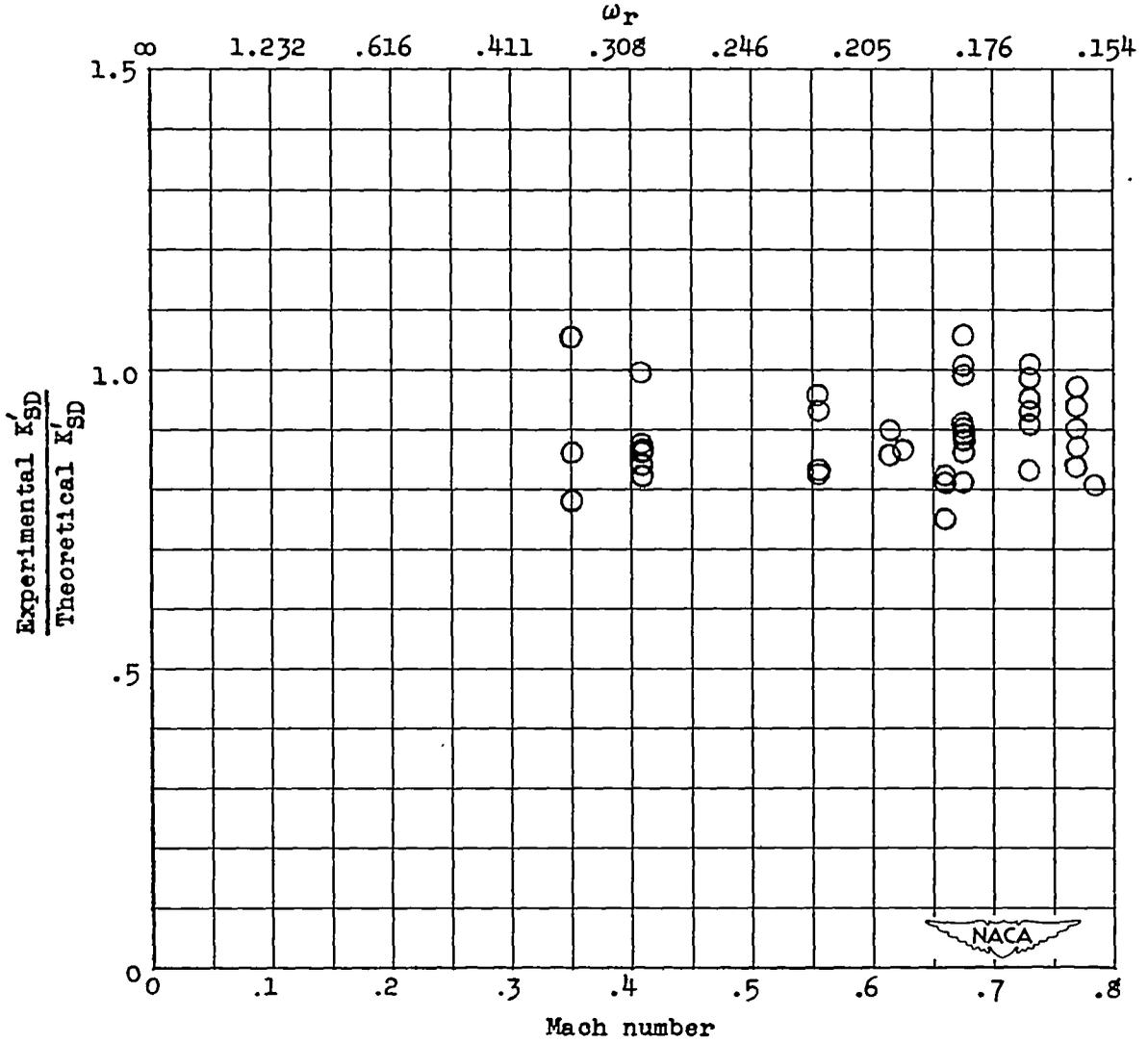


Figure 8.- Ratio of experimental  $K'_{SD}$  to theoretical  $K'_{SD}$  against Mach number and reduced frequency for wing-rod 1.

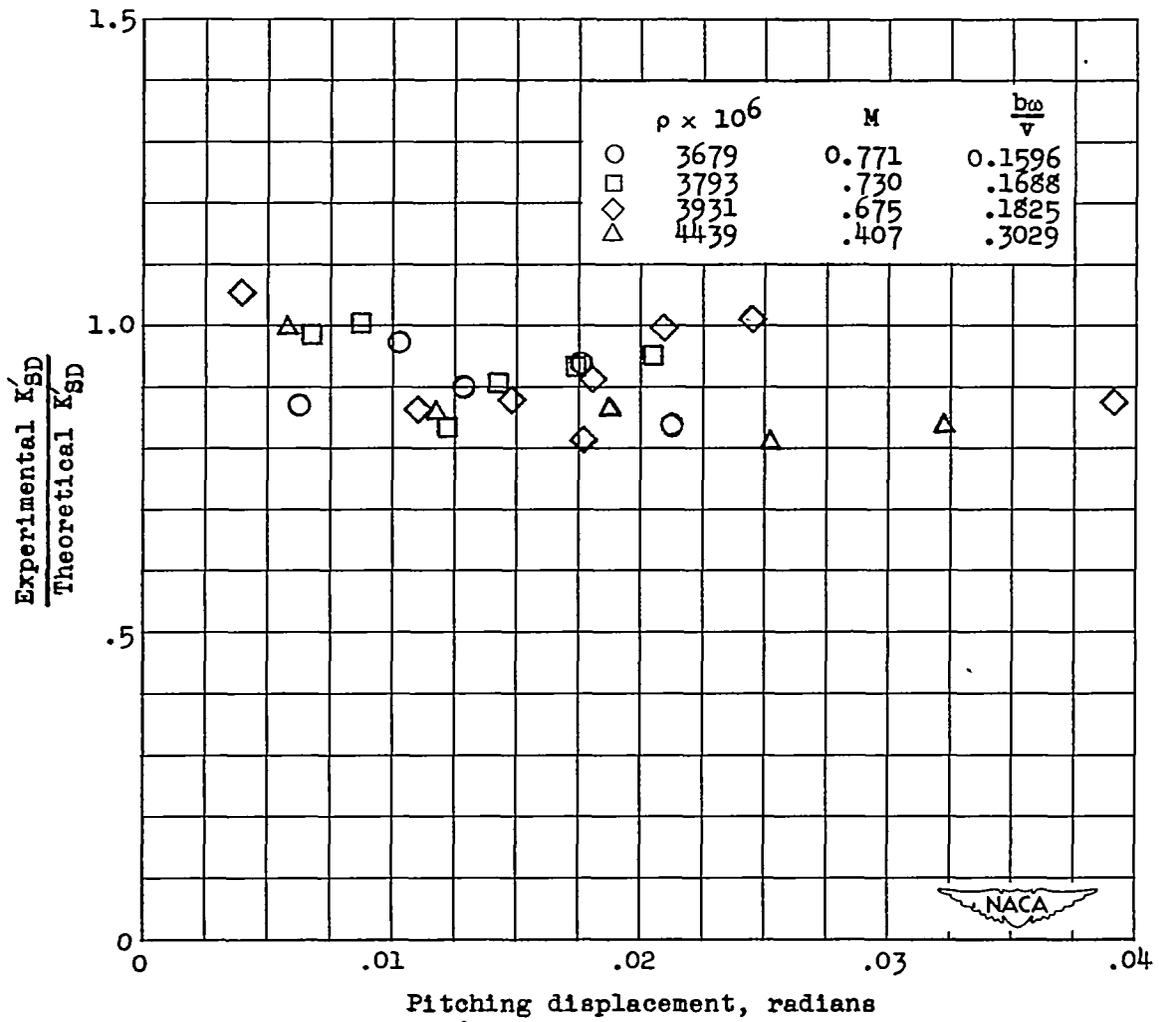


Figure 9.— Ratio of experimental  $K'_{SD}$  to theoretical  $K'_{SD}$  against pitching displacement for wing-rod 1.

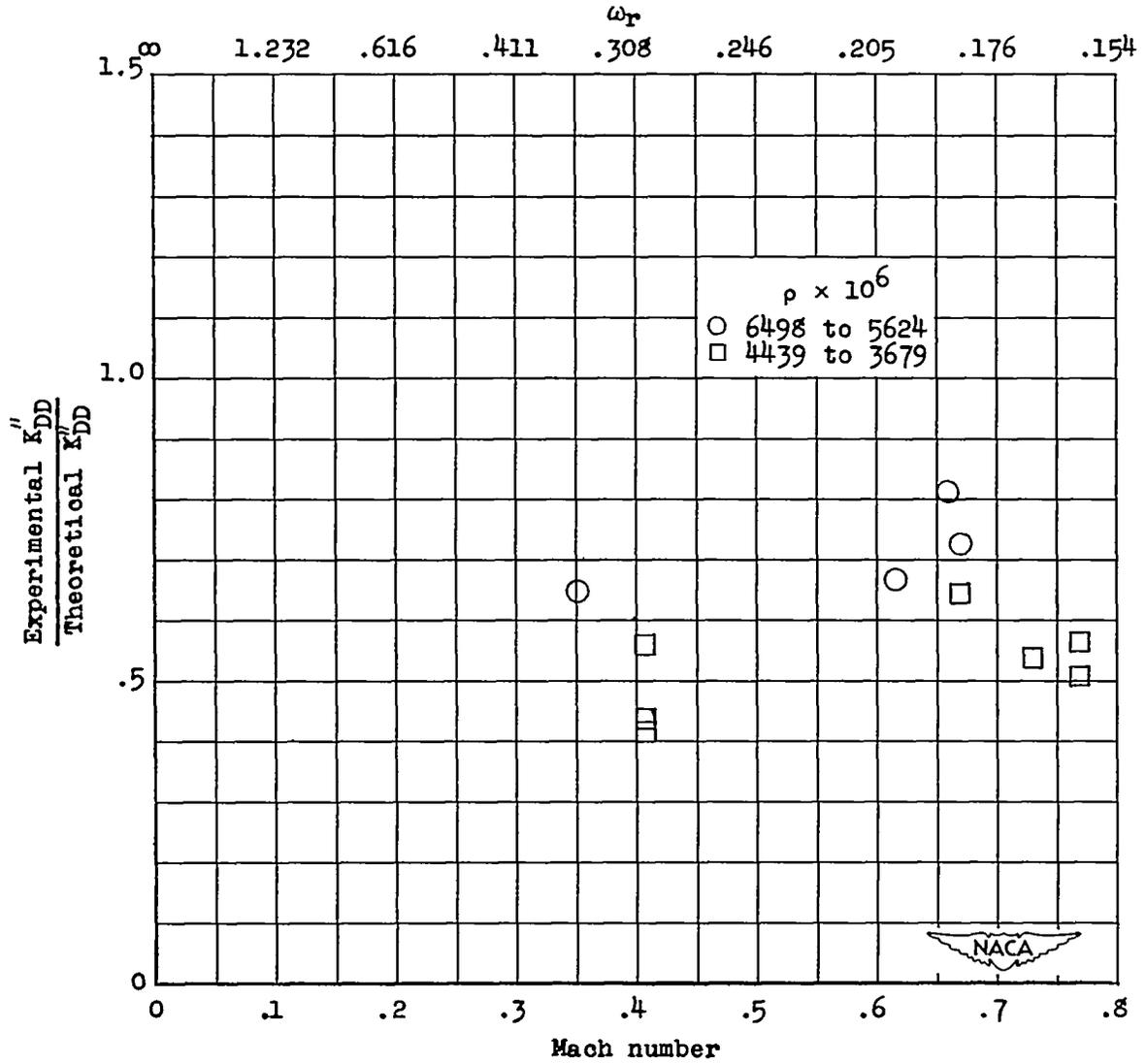


Figure 10.— Ratio of experimental  $K''_{DD}$  to theoretical  $K''_{DD}$  against Mach number and reduced frequency for wing-rod 1.

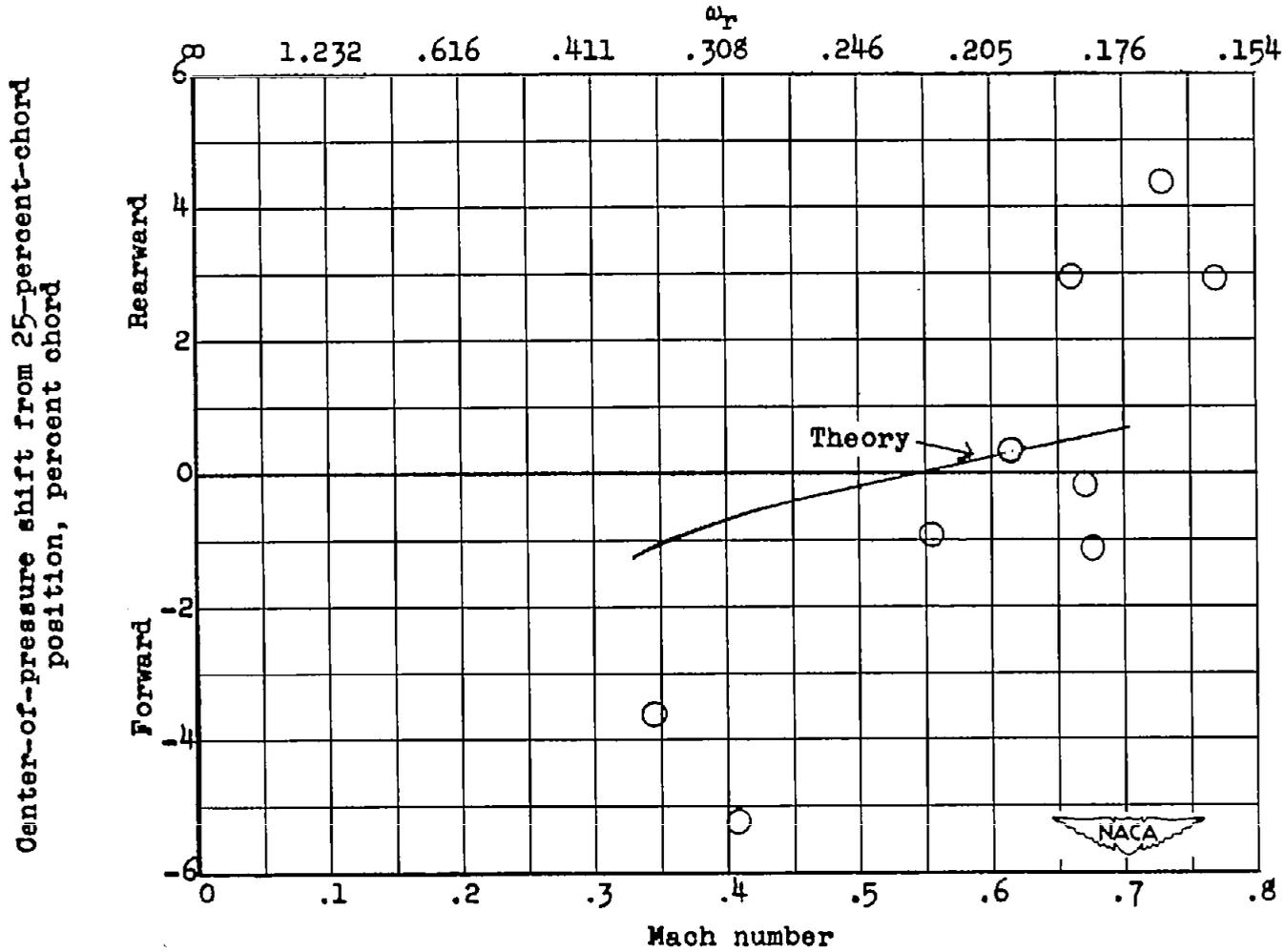


Figure 11.- Experimental and theoretical center-of-pressure shift from the 25-percent-chord position against Mach number and reduced frequency for wing-rod 1.